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SOLUTION OF A THREE-DIMENSIONAL BOUNDARY-VALUE PROBLEM FOR A FRACTIONAL DIFFERENTIAL HEAT CONDUCTION EQUATION

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We consider a three-dimensional axisymmetric problem for a differential heat conduction equation with fractional time derivatives. Using the method of homogeneous solutions and integral transformations, we obtain an asymptotic and a numerical solution of the problem. The results of calculations are presented.

In recent years, the interest of researchers in the physicomechanical properties of materials with a fractal and multifractal structure has grown [5, 9]. Highly porous structured media, which are the result of many processes and phenomena of irreversible growth such as diffusion, aggregation, fracture, percolation, dynamic chaos, and dissolution, can serve as examples of fractals. In the literature, there exist several models for description of the mechanical, thermal, and electric properties of fractal media [5, 10]. They contain partial differential equation with fractional derivatives (with respect to space variables). The fractional differentiation index is related to the fractal dimensionality of the space. However, along with geometrical fractals, time fractals are also considered because there exist processes that have fractal nature in time. Among them are processes of charge transfer in amorphous media within the framework of the random-walk model with continuous time, impact wave processes in condensed media, anomalous diffusion, and heat conduction [9–14].

At present, in the literature, there are several approaches to the solution of differential equations with fractional derivatives. In [1], the Monte Carlo method in combination with the scheme of finite differences was used. The methods of integral transformations are efficient [10]. In [13], the problem of anomalous diffusion in an infinite medium with a cylindrical cavity was considered. Some general reasoning on the solution of fractional differential equations were presented in [11].

In the present work, we propose analytic and numerical procedures for the solution of a new three-dimensional boundary-value problem of fractal heat conduction for a layer with a through cylindrical cavity. Numerical results that characterize the evolution of temperature with time for different orders of the derivative under the action of pulse load on the surface of the body are presented.

Statement of the Problem

In a curvilinear Cartesian system $Ox_1x_2x_3$, we consider a layer $-\infty < x_1, x_2 < \infty$, $|x_3| \le h$, containing a through cylindrical cavity $-h \le x_3 \le h$, $0 < \rho \le R$. We assume that a heat flux is given on the surface of the cavity and zero temperature is maintained on the base surfaces of the layer.

The system of equations that describes the solution of the stated problem has the form

– heat conduction equation [12]

$$\frac{\partial^\alpha T}{\partial t^\alpha} = a^2 \Delta T, \qquad 0 < \alpha \le 2; \tag{1}$$

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